

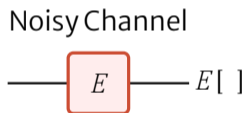


Resource Theories and Noise Reduction

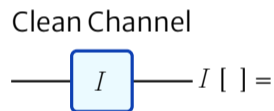
Graeme Berk, Simon Milz, Felix Pollock, and Kavan Modi
5th of December 2022

1. **Noise Reduction** as a Resource Transformation
2. **Multitime Processes** as Resources
3. **Resource Theories** for Noise Reduction
4. **Numerical Results**
5. **Bounding Noise Reduction**

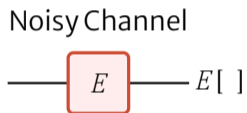
Noise Reduction as a Resource Transformation



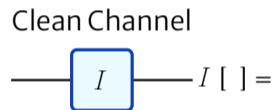
Noise Reduction
7!



Noise Reduction as a Resource Transformation

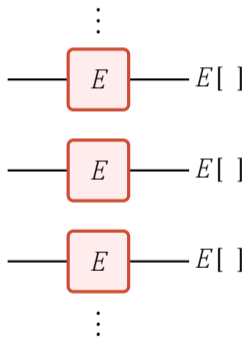


Noise Reduction
7!



Golden rule for quantum resource theories: free transformations cannot increase resource value.

Noise Reduction as Resource Distillation



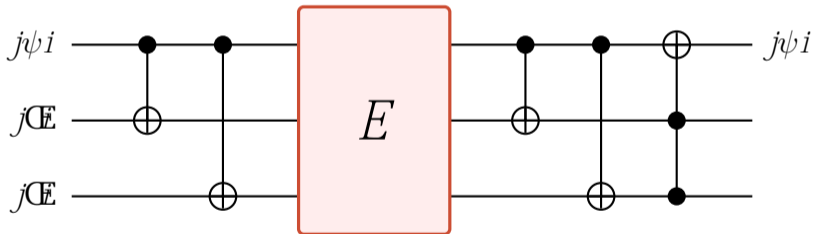
Distillation

7!

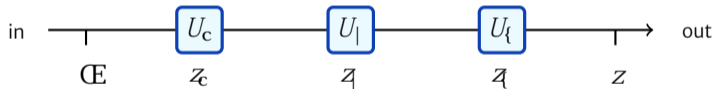


Noise Reduction as Resource Distillation

Bit flip code

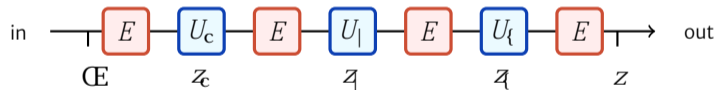


Intermediate interventions to reduce noise



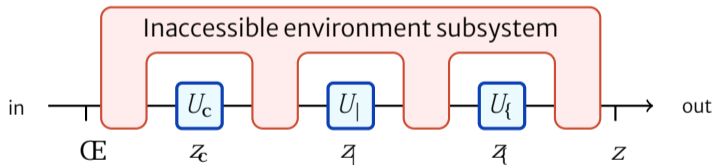
Multi-Time Processes

Model noise as sequence of channels?

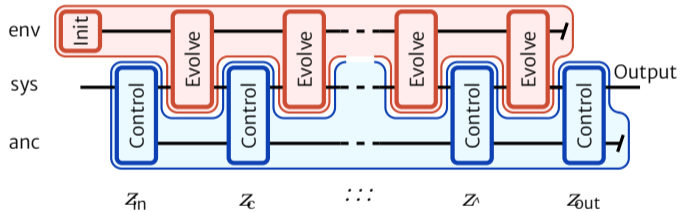


Multi-Time Processes

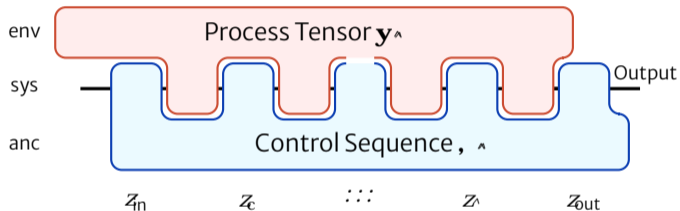
Model noise as a multitime object



Multi-Time Processes



Multi-Time Processes

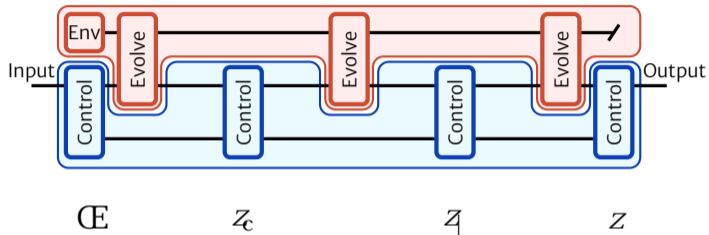


Contracting tensors: $s_{out} := J \mathbf{y}^\wedge j, \wedge K; \wedge M = f z_c; \dots; z^\wedge g$

New Kind of Dynamical Resource Theory

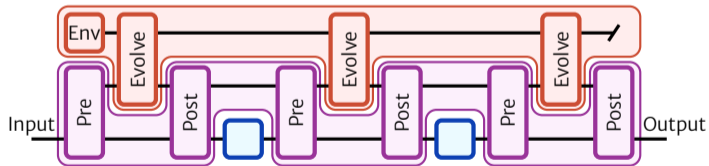
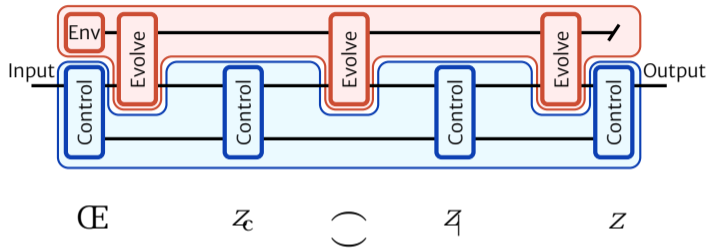
- Resource Objects: Process Tensors
- Resource Transformations: ???

Superprocesses



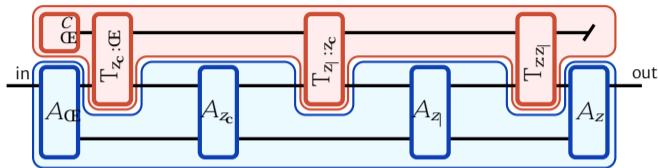
G. D. Berk, A. J. P. Garner, B. Yadin, K. Modi, and F. A. Pollock, **Quantum** (2021).

Superprocesses

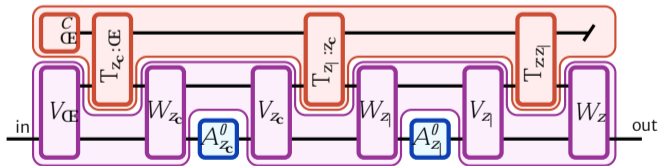


G. D. Berk, A. J. P. Garner, B. Yadin, K. Modi, and F. A. Pollock, **Quantum** (2021).

Superprocesses

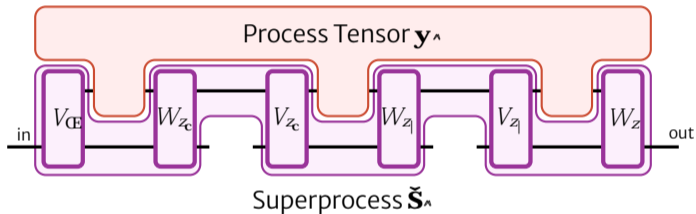


$$J_{\mathbf{y}_{\hat{n}j}, \hat{n}^K} = J_{\mathbf{y}_{\hat{n}j}} \mathbf{S}_{\hat{n}j, \hat{n}^K}$$



G. D. Berk, A. J. P. Garner, B. Yadin, K. Modi, and F. A. Pollock, **Quantum** (2021).

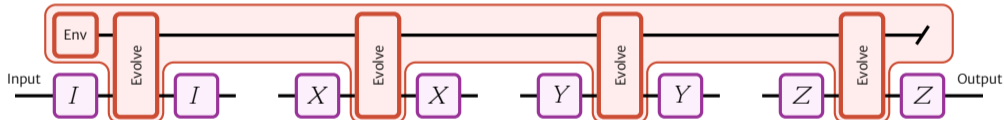
Superprocesses



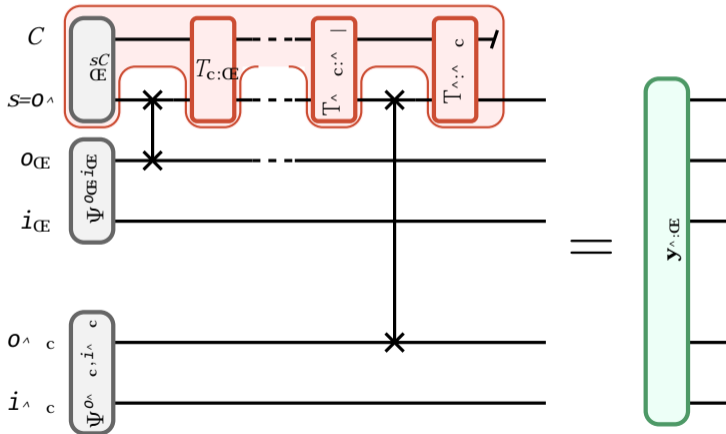
Action of Superprocess: $\mathbf{y}_\lambda^0 := J\mathbf{y}_\lambda j\mathbf{S}^\lambda$

Dynamical Decoupling as a Superprocess

Dynamical decoupling can be cast as a specific kind of superprocess.



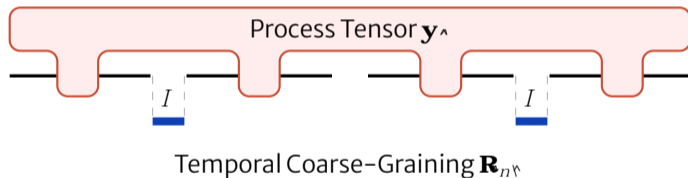
Choi Isomorphism: multitime process, multipartite state



We can concentrate resources amongst **temporal** subsystems!

Temporal Coarse-Graining

Second type of resource transformation



Action of Coarse-Graining: $\mathbf{y}_{\uparrow} := \mathbf{J} \mathbf{y}^\wedge \mathbf{J} \mathbf{R}_{n\uparrow}$; $\uparrow \quad \wedge$

RT for Noise Reduction: Independent Quantum Instruments

Potential Resources: Arbitrary process tensors \mathbf{y}^Λ .

Free Transformations: $\check{\mathbf{S}}^\Lambda \in \mathbb{R}_{n \times n}^{\mathbb{K}}$ consist of superprocesses and/or temporal coarse-graining.

Constraints: free superprocesses are temporally local sequences of quantum operations

$$\check{\mathbf{S}}^\Lambda = \bigcirc_{S \in \mathcal{C}} W_{Z_S} \quad ! \quad V_{Z_S} \quad V_{\mathcal{C}'}^{\dagger}$$

Monotones of IQI

Total mutual information R is a monotone.

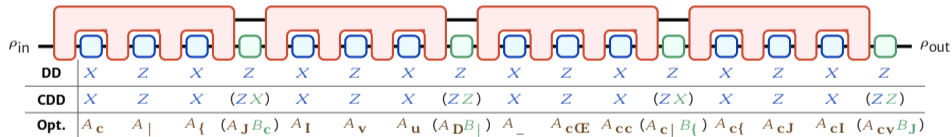
$$R(\mathbf{y}^\wedge) = r(\mathbf{y}^\wedge) + k(\mathbf{y}^\wedge) \quad \text{with } \mathbf{y}^{\text{marg}} := \underset{W \in \mathcal{C}}{\arg \max} \text{tr}_W \mathbf{y}^\wedge g;$$

R can be split into two parts: $R(\mathbf{y}^\wedge) = I(\mathbf{y}^\wedge) + J(\mathbf{y}^\wedge)$.

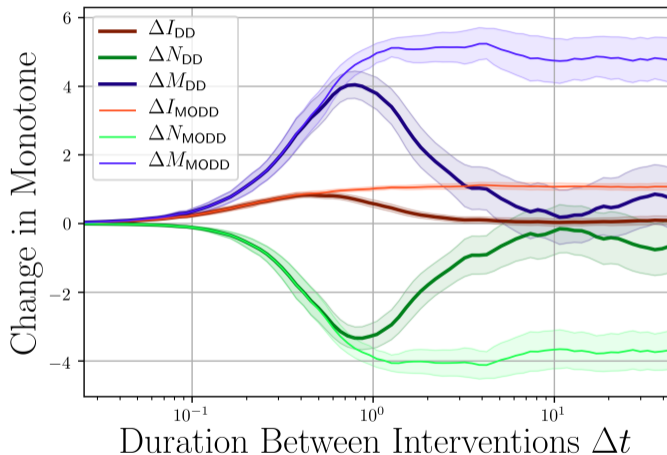
IQI is non-convex, but its monotones **require no optimisation** to compute.

Can We Improve Dynamical Decoupling?

Multiscale Optimal Dynamical Decoupling (MODD) tailors DD sequences to be applied at every available timescale.



Conversion of Non-Markovianity

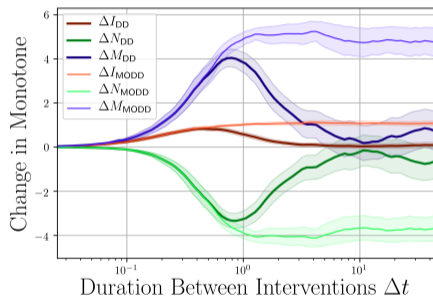


G. D. Berk, S. Milz, F. A. Pollock, and K. Modi, [arXiv:2110.02613](https://arxiv.org/abs/2110.02613) (2021).

Preservation of Information at the Channel-Level

G. D. Berk, S. Milz, F. A. Pollock, and K. Modi, [arXiv:2110.02613](https://arxiv.org/abs/2110.02613) (2021).

Preservation of Information at the Channel-Level



Convex RT: Entanglement Breaking Quantum Instruments

Free superprocesses of IQI:

$$\check{\mathbf{S}}_{\mathcal{A}} = \sum_{\mathcal{S}=\mathcal{C}} \int_{\mathcal{O}} W_{Z\mathcal{S}} \circledast V_{Z\mathcal{S}} \circledast V_{\mathcal{C}\mathcal{E}}$$

Convex combinations of trace non-increasing combs:

$$\check{\mathbf{S}}_{\mathcal{A}} = \sum_W \int_{\mathcal{O}} e_W W_{Z\mathcal{S}_W} \circledast V_{Z\mathcal{S}_W} \circledast V_{\mathcal{C}\mathcal{E}_W}$$

Entanglement **in time** is the resource in EBQI.

Bounding Noise Reduction

Define another theory $\text{ARNG}_{\text{EBQI}}$ using free superprocesses that are **asymptotically resource non-generating** w.r.t. the free processes of EBQI.

Bound on noise reduction:

$$q(\mathbf{y}^\wedge \mid \mathbf{y}^\theta_\wedge) = \frac{r_{\text{RF}}^I(\mathbf{y}^\wedge)}{r_{\text{RF}}^I(\mathbf{y}^\theta_\wedge)};$$

Issues with tightness because permutations of **temporally separated** subsystems are disallowed.

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